Unit 11
Properties of and reasoning about numbers

Five daily lessons

National
Numeracy *Strategy*

Year 6
Spring term

Unit Objectives
Year 6

• Make general statements about odd or even numbers, including the outcome of products.	Page 19
• Know and apply simple tests of divisibility. Find simple common multiples.	Page 19
• Recognise prime numbers to at least 20.	Page 21
• Factorise numbers to 100 into prime factors.	Page 21
• Explain methods and reasoning, orally and in writing.	Page 77
• Solve mathematical problems or puzzles, recognise and explain patterns and relationships, generalise and predict. Suggest extensions asking 'What if . . .?'	Page 79
• Develop from explaining a generalised relationship in words to expressing it in a formula using letters as symbols (e.g. the cost of *n* articles at 15p each).	Page 81

Link Objectives

Year 5

- Make general statements about odd and even numbers.
- Recognise multiples of 6, 7, 8, 9 up to the tenth multiple and know tests of divisibility by 2, 4, 5, 10 or 100.
- Find all the pairs of factors of any number up to 100.

Year 7

- Recognise and use multiples, factors (divisors), highest common factor, lowest common multiple and primes.
- Use simple tests of divisibility.
- Recognise the first few triangular numbers, squares of numbers to at least 12 × 12 and the corresponding roots.
- **Solve word problems and investigate in a range of contexts.**

(Key objectives in bold)

This Unit Plan is designed to guide your teaching.

You will need to adapt it to meet the needs of your class.

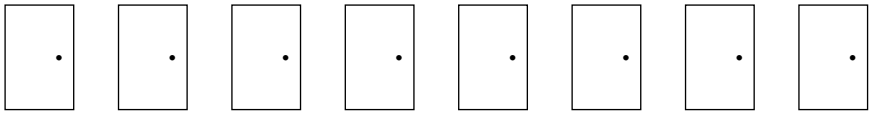
Resources needed to teach this unit:

- Resource sheet 11.1
- Activity sheet 11.1
- Empty tin and cubes
- Whiteboards
- Set of large digit cards 1–9
- OHP calculator
- Large sheet of paper for poster
- Related Key Stage 2 National test questions

department for
education and skills

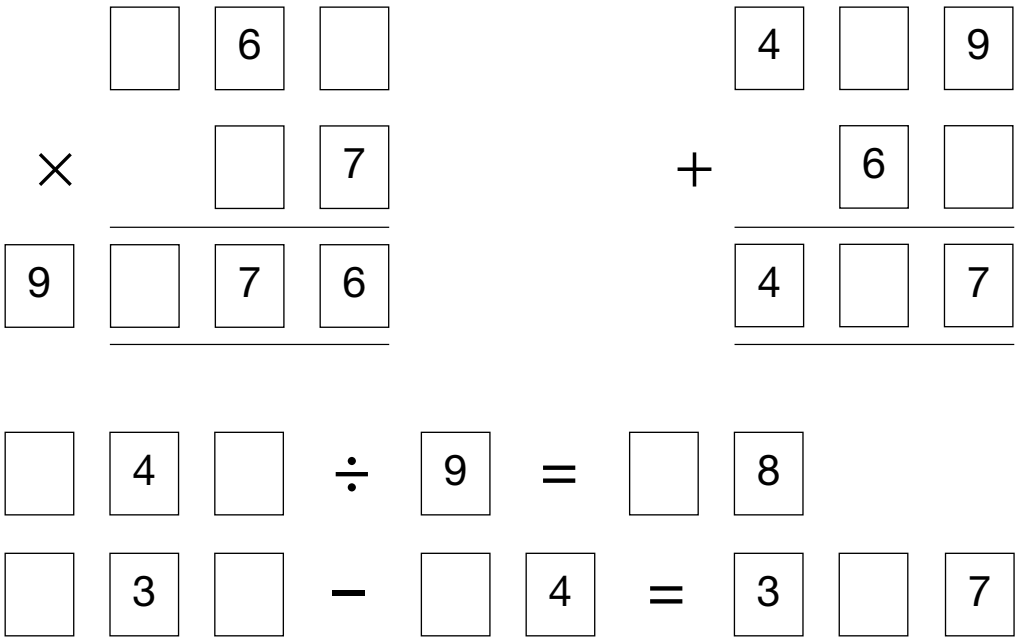
24	72	12	32	56
112	60	40	64	88
120	104	96	84	108

1. There are eight doors in a row. Behind each door is a number. Each number is less than 20. No two numbers are the same, and the numbers on the doors are in order.



Three of the numbers are prime numbers.
Two of the numbers are square numbers.
Two of the numbers are multiples of 5.
Five of the numbers are even.
Three is a factor of two of the numbers.
There are a pair of consecutive numbers, and four consecutive numbers on the doors.
The numbers sum to 93.
What are the numbers behind the doors?

2. Write in the missing digits:



Related Key Stage 2 national test questions:

2001 Test A

9 Write in the **missing** digits to make this correct.

$$\begin{array}{rcccc} & & \square & 4 & \square \\ \times & & & & 6 \\ \hline & 2 & 0 & 5 & 2 \\ \hline \end{array}$$

9a

1 mark

9b

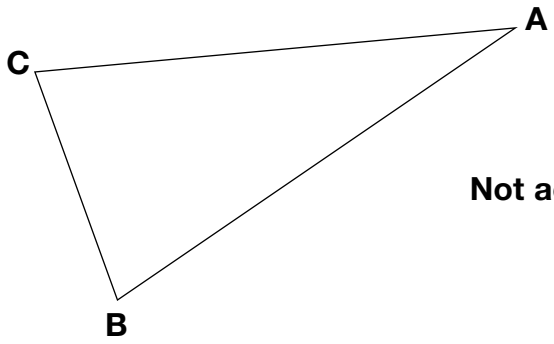
1 mark

Total

2001 Test A

21 Triangle **ABC** is isosceles and has a perimeter of 20 centimetres.

Sides **AB** and **AC** are each **twice** as long as **BC.**



Not actual size

Calculate the length of the side **BC**.

Do **not** use a ruler.

Show your **working.** You may get a mark.

cm

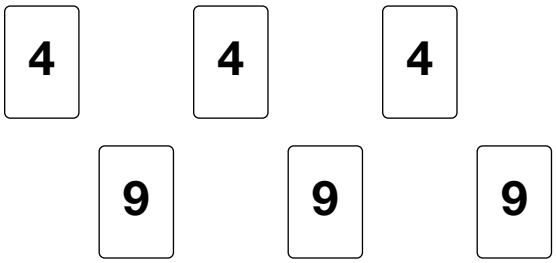
21

2 marks

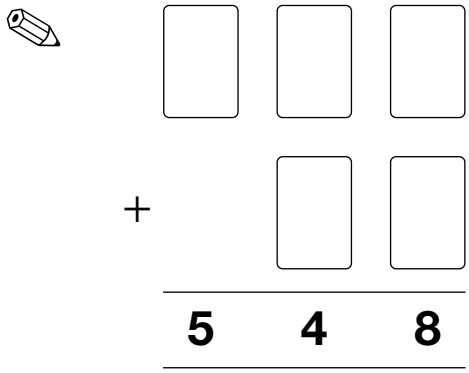
Total

2001 Test B

6 Here are some number cards.



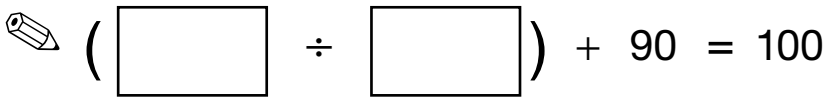
Use **five of the number cards** to make this correct.



6

2 marks

7 Write in what the **missing** numbers could be.



7

1 mark

Total

2001 Test B

23 Write the **three prime numbers** which multiply to make **231**.

$\square \times \square \times \square = 231$

23

1 mark

Total

2000 Test A

2 Write in the **missing** numbers.

150 + ☐ = 500

2a

1 mark

172 − ☐ = 60

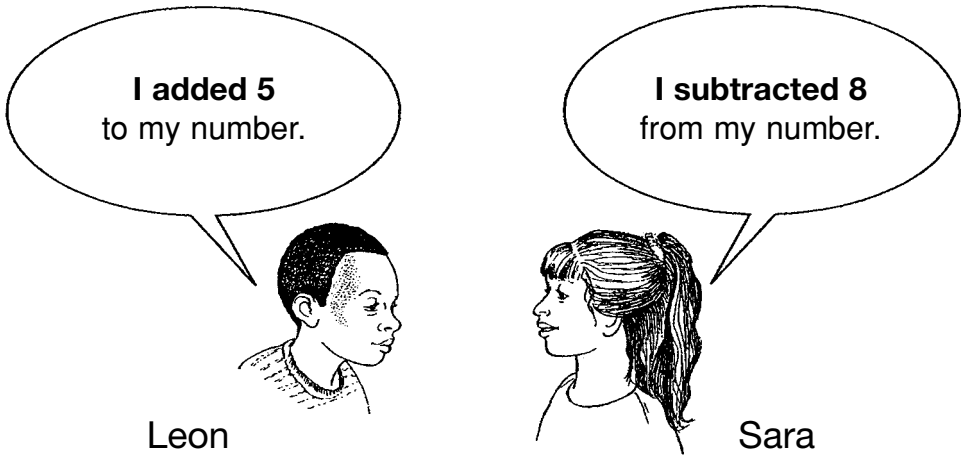
2b

1 mark

Total

2000 Test A

12 Leon and Sara each started with **different** numbers.



Leon and Sara both get the **same** answer.

What numbers could they have started with?

Leon ☐ Sara ☐

12

1 mark

Total

2000 Test B

4 Write in the missing number.

$60 + 99 + \square = 340$

4

1 mark

Total

2000 Test B

7 Write **two numbers,** each **greater than 100,** to complete this subtraction.

			−				=	2	0	8

7

1 mark

Total

2000 Test B

12 Nadia is working with **whole** numbers.

She says,

'If you add a two-digit number to a two-digit number you cannot get a four-digit number.'

Is she correct? Circle Yes or No. Yes / No

Explain why.

.....

.....

.....

12

1 mark

Total

2000 Test B

18 n stands for a number.

Complete this table of values.

n	$5n - 2$
20	
	38

18a 1 mark

18b 1 mark

Total

2001 Test B

20 A sequence starts at **500** and **80** is **subtracted** each time.

500 420 340 ...

The sequence continues in the same way.

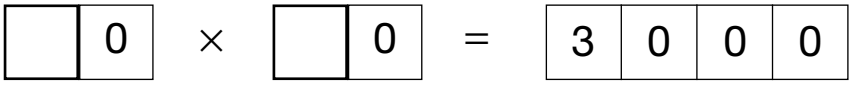
Write the **first two numbers** in the sequence which are **less than zero.**

20

2 marks

2002 Test A

19 Write in the **two** missing digits.



19

1 mark

Total

2002 Mental Arithmetic

18	Look at your answer sheet. Put a ring around the number which is **not** a factor of three hundred.	60	75	90	100	150

13	Ten times a number is eighty-six. What is the number?		86

Planning sheet	Day One	Unit 11	*Properties of and reasoning about numbers*	Term: *Spring*	Year Group: *6*

Oral and Mental

Objectives and Vocabulary

Count on and back in steps of 7 and 8.

Recall multiplication and division facts to 10×10.

RESOURCES
Empty tin and cubes

Teaching Activities

- Divide the class into 4 groups. Start one group counting in steps of 7; starting from 7. Point to a different group to continue the count. Count to beyond 70 and involve each group. Repeat, this time say 'backwards' or 'forwards' to a group. Extend into negative numbers. Repeat for 8s.
- Use a tin and some cubes.

  Explain to children that as they hear each cube drop into the tin they should count on 7 silently with their eyes closed.

  Q What was the last number you reached?
  Q How may 7s did you count?
  Q How many cubes are now in the tin?
  Q How many 7s did you count?
  Q Which multiplication fact is this?
  Q Which division fact is this?

  Repeat activity for 8s.

Main Teaching

Objectives and Vocabulary

Make general statements about odd and even numbers including the outcome of products.

Find simple common multiples.

VOCABULARY
odd, even
product
multiple of
common multiple
lowest common multiple

RESOURCES
Resource sheet 11.1

Teaching Activities

- Ask a child to provide you with two even numbers less than 10 (say, 4 and 8) and then to give you the product. Write $4 \times 8 = 32$ on the board. Repeat using other pairs of even numbers.

  Q What pattern can you see in the results?

  Establish that the product of two even single-digit numbers appears to be even.

  Q If we did this for all pairs of single-digit even numbers would it be true for all cases?

- Repeat using two single-digit odd numbers, and finally, one odd and one even single-digit numbers.

  Q What statements can we make for pairs of single-digit numbers?

  Collect answers and record on the board: 'For pairs of single-digit numbers:

  even $\times$ even = even; odd $\times$ even = even;
  even $\times$ odd = even; odd $\times$ odd = odd'

  Q Can you explain why these general statements must be true for all products of two numbers?

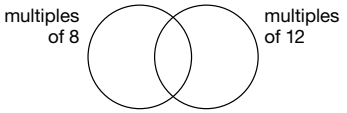
  Collect children's responses and arguments. Draw out that whether a number is odd or even depends on its units digit. When we multiply two numbers together the product of the units digits determines the units digit in the answer. Write on the board:

  $$73\,924 \times 60\,399$$

  Q Will the answer be odd or even?

  Ensure children recognise the units digit in the answer is 6 as $4 \times 9 = 36$. Repeat with other pairs of numbers.

- Draw on the board:

  Ask children to explain the diagram. Remind them it is called a Venn diagram. Write 16, 36, 48 on the board.

  Q Where would we put these numbers in the Venn diagram?

  Establish children can do this correctly. Display Resource sheet 11.1. Get children to work in pairs putting the numbers onto the Venn Diagram.
  Collect answers adding them to the diagram on the board.

  Q How could we describe the overlapping section?

  Establish that all these numbers are multiples of 8 and 12.

  Q What is the smallest number that is a multiple of 8 and 12?

  Establish from the Venn diagram this is 24. Say that 24 is the lowest common multiple of 8 and 12. All the other numbers in the overlapping section are also common multiples of 8 and 12.

  Q What numbers are common multiples of 6 and 9?

  Collect answers and agree that the lowest common multiple of 6 and 9 is 18 and all other common multiples will be multiples of 18. Repeat for other pairs e.g. 4 and 10, 6 and 15, 12 and 16. Discuss answers and strategies. Demonstrate how to find common multiples using lists of the multiples of each number.

Plenary

Teaching Activities / Focus Questions

- **Q** How can we find common multiples of 15 and 18?

  Use lists to find the lowest common multiple is 90, then find multiples of 90.

  Repeat for 12 and 20.

By the end of the lesson the children should be able to:

- **Find common multiples of two numbers such as 12 and 16;**
- **Recognise odd and even numbers and make general statements such as 'the product of one odd and one even number is even';**
- **Find common multiples of two numbers such as 8 and 12;**
- **Find the smallest number that is a common multiple of two numbers such as 6 and 15.**

(Refer to supplement of examples, section 6, page 19.)

Planning sheet	Day Two	Unit 11	*Properties of and reasoning about numbers*	Term: *Spring*	Year Group: *6*

Oral and Mental

Objectives and Vocabulary	Teaching Activities
Recognise multiples to at least 10 × 10. VOCABULARY common multiple RESOURCES Whiteboards	• Using whiteboards to show their answers, children respond to questions such as: Show me: – a multiple of 5 . . . – a multiple of 6 that is greater than 30 – a common multiple of 3 and 7 – a number which is a multiple of 6 but between 70 and 100. • Ask children to write any two-digit number on their whiteboards. Call out questions such as: Show me: – a multiple of 7 – a common multiple of 6 and 4 – one more than a multiple of 3 – a prime number – a square number. Use a range of questions. Pupils hold up whiteboards if their number matches the statement.

Main Teaching

Objectives and Vocabulary	Teaching Activities
Know and use tests of divisibility VOCABULARY divisible by divisibility sum of the digits multiple of common multiple RESOURCES Large sheet of paper for making poster OHP calculator	• Begin by saying 'I am counting in 10s from zero. Will I say 139? How do you know?' **Q** Is 139 divisible by 10? What's the rule? Record the rule, on large sheet of paper for display. • Repeat with counting in: 2s **Q** Is 2241 divisible by 2? 4s **Q** Is 57 716 divisible by 4? 5s **Q** Is 10 005 divisible by 5? 25s **Q** Is 32 275 divisible by 25? In each case agree a rule and record them on the large sheet. **Q** Can we find a rule for 8s? Build on the rules for 2 and 4 to establish that the last three digits of the number must form a multiple of 8. Ask the class to recite the multiples of 9 up to 100. On the board write these multiples of 9. **Q** What can you say about the sum of the digits in each multiple? Discuss and establish the test: a whole number is divisible by 9 if the sum of its digits is divisible by 9. Ask the children to write four-digit numbers that are multiples of 9. Apply the test to a sample of numbers and confirm with OHP calculator. • Using a similar process establish the test of divisibility by 3 (the sum of the digits is divisible by 3) and link to test for divisibility by 6 (the number is even and divisible by 3). Add these rules to the display. • Go through the rules again. Say that we have tests of divisibility for 2, 3, 4, 5, 6, 8, 9, 10. Children should remember them and use them when multiplying and dividing. • Note that we do not have a test of divisibility for 7. **Q** Is 1249 divisible by 7? Write on the board: 1249 = 1200 + 49 Say if 1249 is a multiple of 7 then 1200 and 49 must be too. Agree that 49 is but 1200 is not. Confirm 1249 is not divisible by 7 on the OHP calculator. Explain that this partitioning approach will always work. **Q** Is 3521 divisible by 7? Agree that this is because 3500 and 21 are. Confirm on the OHP calculator.

Plenary

Teaching Activities / Focus Questions
• On the board write: 123 456. Ask children to apply the tests of divisibility for 2, 3, 4, 5, 6, 7, 8 and 9 to this number. Correct answers and confirm each case with the OHP calculator. • Repeat for 561 432. Establish that as this has the same digits as 123 456, we know some of the tests apply. Emphasise that tests of divisibility can help to check calculations. **Q** Is 165 × 9 = 1495 correct? Remind children that, in the last lesson we found that 165 × 9 would end in 5 but 1 + 4 + 9 + 5 = 19 so 1495 is not a multiple of 9. So the multiplication is incorrect. Repeat using other multiplication statements such as: **Q** Is 214 × 6 = 1324 correct? HOMEWORK – Give children the following problems to solve: □ □ 8 ÷ 6 = 4 □ 3 □ □ ÷ 4 = □ 9 14 □ ÷ □ = □ 8 Explain each box represents a digit. Remind children to use tests of divisibility to help solve the problem **By the end of the lesson the children should be able to:** • **Recognise when a whole number is divisible by 2, 3, 4, 5, 6, 8, 9, 10 and 25.** (Refer to supplement of examples, section 6, pages 19 and 73.)

Planning sheet	Day Three	Unit 11	*Properties of and reasoning about numbers*	Term: *Spring*	Year Group: *6*

Oral and Mental

Objectives and Vocabulary

Use tests of divisibility.

RESOURCES
Set of large digit cards 1 to 9
Whiteboards

Teaching Activities

- Write on the board:

  2, 3, 4, 5, 6, 7, 8, 9, 10.

  Quickly discuss the tests of divisibility for each number.

- Shuffle the set of digit cards. Turn over the top three cards and attach them to the board to form a three-digit number, e.g. 714.

  Children to apply tests of divisibility and show using whiteboards which of the numbers 2 to 10 divide into 714.

  Repeat.

- Extend using four cards to make a four-digit number.

Main Teaching

Objectives and Vocabulary

Recognise prime numbers to at least 20.

Factorise numbers to 100 into prime factors.

Find simple common multiples and factors.

VOCABULARY
prime number
factor
prime factor
factorise

Teaching Activities

- Remind children of the meanings of 'factor' and 'multiple' by considering a number such as 14. 14 is a *multiple* of 2 and 7; 2 and 7 are *factors* of 14.

  Ask children for factors and multiples.

  Q Is 9 a factor for 360?
  Q Is 745 a multiple of 5?

- Ask children to recite the prime numbers to 20 starting with 2. Remind them that a prime number has two factors only.

  Q Is 1 a prime number?

  Agree that it is not as it has only one factor.

- Say that 7 is a prime factor of 35. Ask children why this is so. Agree that 7 is a factor of 35 and 7 is prime, so 7 is a prime factor of 35.

  Q What numbers are prime factors of 50?

  Agree that 50 has two prime factors, 2 and 5.

  Q How is 50 made up of its prime factors?

  Establish that $50 = 2 \times 5 \times 5$.

- Work through some examples to show how to represent numbers as products of their prime factors:

  e.g. $12 = 4 \times 3$ $\quad 45 = 9 \times 5$
  $12 = 2 \times 2 \times 3$ $\quad 45 = 3 \times 3 \times 5$

- Ask children to choose numbers less than 100 and to represent them as products of their prime factors. Collect examples and correct any errors or misunderstandings.

- Explain that representing numbers as their products of prime factors helps to find common factors and common multiples. Write on the board 24 and 18.

  With the class work through

  $24 = 2 \times 12$ $\quad 18 = 2 \times 9$
  $= 2 \times 2 \times 6$ $\quad = 2 \times 3 \times 3$
  $= 2 \times 2 \times 2 \times 3$

  Q What prime factors do 24 and 18 have in common?

  Agree they are 2 or 3. Explain that this means $2 \times 3 = 6$ is the highest common factor number that divides into 24 and 18. Write:

  $24 = 2 \times 2 \times \boxed{\begin{matrix} 2 \times 3 \\ 2 \times 3 \end{matrix}}$
  $18 = \qquad\quad \times 3$

  Explain that as 2×3 appears twice we shall not repeat these factors.

  Q What is $2 \times 2 \times 2 \times 3 \times 3$?

  Agree it is $8 \times 9 = 72$

  Explain that 72 is the smallest number that 24 and 18 divide into, the lowest common multiple.

  On the board work through finding the highest factor and lowest common multiple of 28 and 21.

  $28 = 2 \times 14 = 2 \times 2 \times \boxed{7}$
  $21 = 3 \times 7 = \qquad\quad \boxed{7} \times 3$

  Highest common factor = 7

  Lowest common multiple is

  $2 \times 2 \times 7 \times 3 = 4 \times 21 = 84$

  Set children examples to do. Collect answers and correct errors and misunderstandings.

Plenary

Teaching Activities / Focus Questions

- Discuss the tests of divisibility and how they can be used to find prime numbers.

  Q Is 51 a prime number? How can we be sure?

- Remind children that prime numbers have only two factors – themselves and 1.

  Repeat for 31, 61, 57, 87.

By the end of the lesson the children should be able to:

- **Say which of these are prime numbers: 11, 21, 31, 41, 51, 61;**
- **Recognise prime numbers to at least 20;**
- **Find the prime factors of numbers to 100.**

(Refer to supplement of examples, section 6, page 21.)

Planning sheet	Day Four	Unit 11 *Properties of and reasoning about numbers*	Term: *Spring*	Year Group: *6*

Oral and Mental

Objectives and Vocabulary	Teaching Activities
Derive sums and differences. RESOURCES Whiteboards	• Ask six children each to say a three-digit number. Write them on the board. Tell the children they have to find as many sums and differences of pairs of numbers as they can; within a given time period. **Q** Did anyone find a difference of less than 10? **Q** Did anyone find a total of more than 1500?

Main Teaching

Objectives and Vocabulary	Teaching Activities
Develop a generalised relationship in words; express it in a formula using symbols. VOCABULARY consecutive sum formula symbols	• Write 5, 10, 15 on the board. Ask children to describe these numbers. Establish that they are the first three consecutive multiples of 5. **Q** What is the sum of these numbers? Collect answers and discuss the children's strategies. **Q** Can you propose a short-cut to find their sum? Encourage different suggestions: Three times 10; two times 15; six times 5. **Q** Will these short-cuts always work? Get children to work in pairs, finding the sums of the first three consecutive numbers in different times tables and testing the short-cuts each time. **Q** Did they all work? How would you describe the shortcuts for someone not in your class? Encourage children to be precise. Record their suggestions and refine them e.g. for the first three consecutive numbers in a times table, their sum is six times the first number. **Q** Could we use symbols to describe our shortcuts? Suggest the three consecutive numbers are *f, m, l* for first, middle, last. Encourage children to begin to develop formulae. Sum = 6*f*, Sum = 3*m*, Sum = 2*l*. Explain that these are formulae and you now want the children to make up their own formulae for other sums of consecutive numbers. Encourage some children to explore four consecutive numbers.

Plenary

Teaching Activities / Focus Questions

- Give the children the sum of four consecutive numbers, e.g. 70, 120 and challenge them to find the consecutive numbers.

Q What strategy did you use? Why?

By the end of the lesson the children should be able to:

- **Make a general statement about consecutive numbers, find examples that match it and express it in a formula using symbols, as appropriate.**

(Refer to supplement of examples, section 6, page 81.)

Planning sheet	Day Five	Unit 11 *Properties of and reasoning about numbers*	Term: *Spring*	Year Group: *6*

Oral and Mental

Objectives and Vocabulary	Teaching Activities
Recall multiplication and division facts. Use and interpret symbols. VOCABULARY symbol RESOURCES Whiteboards	• Say you are thinking of a number, you double it and add 1. **Q** How could you use symbols to describe this? Agree that this is the form $2n + 1$. Use a variety of symbols. Repeat giving different descriptions of operations for children to write down on whiteboards using symbols. • Write on the board: $3h - 1 = 8$ **Q** What is h? Children to write answers on whiteboards. Collect responses and discuss strategies. Repeat using similar questions, using different letters and symbols, e.g. $5v + 3 = 28$. Encourage children to 'read' the questions – 'I think of a number, multiply it by 5 and add 3 to get 28'. Help them to reduce this by removing the odd 3 to get 'I multiply it by 5 to get 25.' Show children how to record this to help them: $5v + 3 = 28$, $5v = 25$, $v = 5$ Encourage them to jot down any calculations that might help them.

Main Teaching

Objectives and Vocabulary	Teaching Activities
Solve number puzzles and explain methods and reasoning. VOCABULARY product divisible RESOURCES Activity sheet 11.1	• Review the homework from Day Two. Check answers: $258 \div 6 = 43$; $356 \div 4 = 89$; $144 \div 3 = 48$; Discuss the strategies the children used. **Q** What key bits of information helped most? Discuss importance of knowing tables and recognising how to use these facts as clues. • Write on the board: $\boxed{1}\ \square\ \boxed{3} \times \square\ \square = 5535$ **Q** What number facts might help us to solve this problem? Draw out the following and write them on the board: – knowledge of multiplication facts to look at last digits; **Q** Do we know how the second number ends? – the odd and even outcomes of multiplication; – tests of divisibility. **Q** Which tests of divisibility could we use? (3, 9) Discuss how children might use these and work through the problem with the class. (Answer is: 123×45) • Children work on following problem in pairs. $\boxed{9}\ \boxed{6} \times \boxed{8}\ \square = \boxed{7}\ \square\ \square\ \boxed{8}$ Answer is: (96×83) Discuss the clues and strategies used. • Set more puzzles, e.g.: $3\square \times 7\square = 2520$ (35×72) $\square\ \square 6 \times 4\square = 10148$ (236×43) Collect answers and discuss strategies, correct any errors and misunderstandings. • Ask children to invent a problem for their partner, swap these and solve them.

Plenary

Teaching Activities / Focus Questions

- Collect samples of children's own puzzles, work through one or two ensuring that the clues are identified and exemplified.
- Remind children that they have been using different symbols to represent numbers and applying their knowledge of numbers to solve problems. Emphasise how important it is to keep their knowledge of numbers sharp, in readiness for their test next term. Give out Activity sheet 11.1 for children to complete at home.

Answers to Activity sheet 11.1.

1. 2, 4, 10, 11, 15, 16, 17, 18
2. $168 \times 57 = 9576$
   $419 + 68$ or $429 + 68$
   $342 \div 9 = 38$
   $331 - 14 = 317$

By the end of the lesson the children should be able to:

- **Use and explain appropriate strategies for solving number puzzles.**

(Refer to supplement of examples, section 6, page 79.)